

MATHEMATICS
STANDARD LEVEL
PAPER 2

Friday 4 May 2012 (morning)

1 hour 30 minutes

Candidate session number

0	0							
---	---	--	--	--	--	--	--	--

--	--	--	--	--	--	--	--	--	--

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics SL** information booklet is required for this paper.
- The maximum mark for this examination paper is [90 marks].

1. [Maximum mark: 6]

The first three terms of an arithmetic sequence are 36, 40, 44,

(a) (i) Write down the value of d .

(ii) Find u_8 . [3 marks]

(b) (i) Show that $S_n = 2n^2 + 34n$.

(ii) Hence, write down the value of S_{14} . [3 marks]

2. [Maximum mark: 7]

Let $f(x) = 2x^2 - 8x - 9$.

(a) (i) Write down the coordinates of the vertex.

(ii) Hence or otherwise, express the function in the form $f(x) = 2(x - h)^2 + k$. [4 marks]

(b) Solve the equation $f(x) = 0$. [3 marks]

3. [Maximum mark: 6]

Let $\mathbf{M} = \begin{pmatrix} x & 2x \\ x^2 & 1 \end{pmatrix}$ and $\mathbf{N} = \begin{pmatrix} 2 & 3 & -1 \\ 4 & 2 & 0 \\ 1 & 5 & 1 \end{pmatrix}$.

(a) Find $\det \mathbf{M}$.

[2 marks]

(b) Write down $\det \mathbf{N}$.

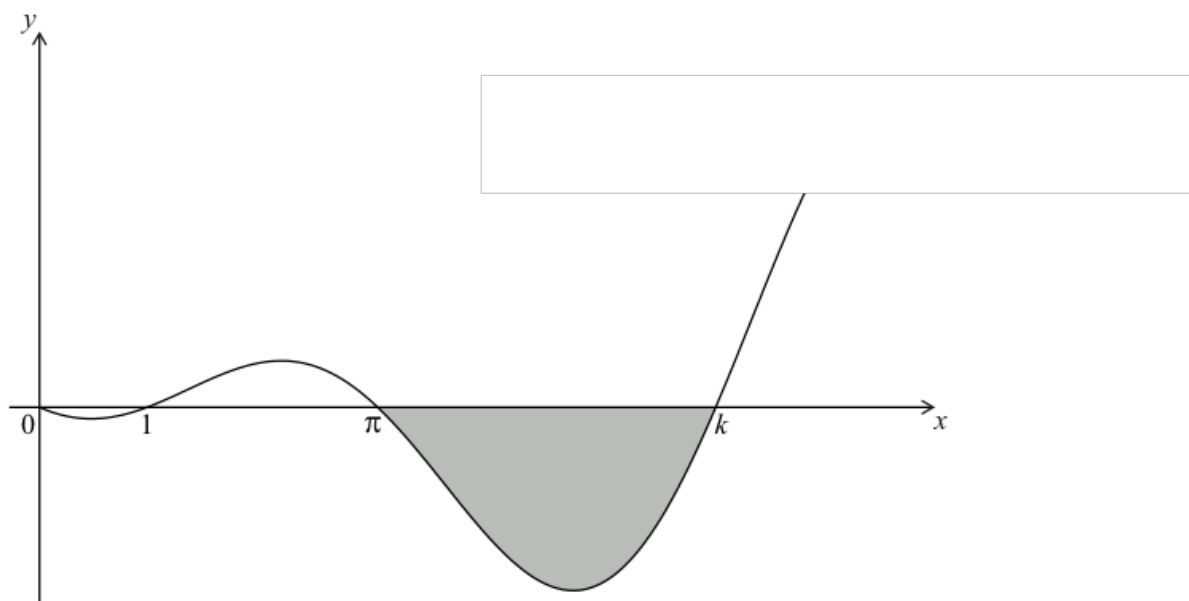
[1 mark]

(c) Find the value of x for which $\det \mathbf{M} = \det \mathbf{N}$.

[3 marks]

4. [Maximum mark: 7]

The graph of $y = (x-1)\sin x$, for $0 \leq x \leq \frac{5\pi}{2}$, is shown below.



The graph has x -intercepts at 0 , 1 , π and k .

(a) Find k . [2 marks]

The shaded region is rotated 360° about the x -axis. Let V be the volume of the solid formed.

(b) Write down an expression for V . [3 marks]

(c) Find V . [2 marks]

5. [Maximum mark: 6]

$$\text{Let } \mathbf{M} = \begin{pmatrix} p & -1 & -2 \\ 1 & 1 & -2 \\ 1 & q & -1 \end{pmatrix} \text{ and } \mathbf{M}^{-1} = \frac{1}{2} \begin{pmatrix} 3 & -5 & 4 \\ -1 & 1 & 0 \\ 1 & -3 & 2 \end{pmatrix}.$$

(a) Find the value of p and of q .

[3 marks]

(b) Solve the system of linear equations.

$$px - y - 2z = 7$$

$$x + y - 2z = 2$$

$$x + qy - z = -3$$

[3 marks]

6. [Maximum mark: 6]

Consider the expansion of $\left(2x^3 + \frac{b}{x}\right)^8 = 256x^{24} + 3072x^{20} + \dots + kx^0 + \dots$

(a) Find b .

[3 marks]

(b) Find k .

[3 marks]

7. *[Maximum mark: 7]*

The probability of obtaining “tails” when a biased coin is tossed is 0.57. The coin is tossed ten times. Find the probability of obtaining

(a) **at least** four tails;

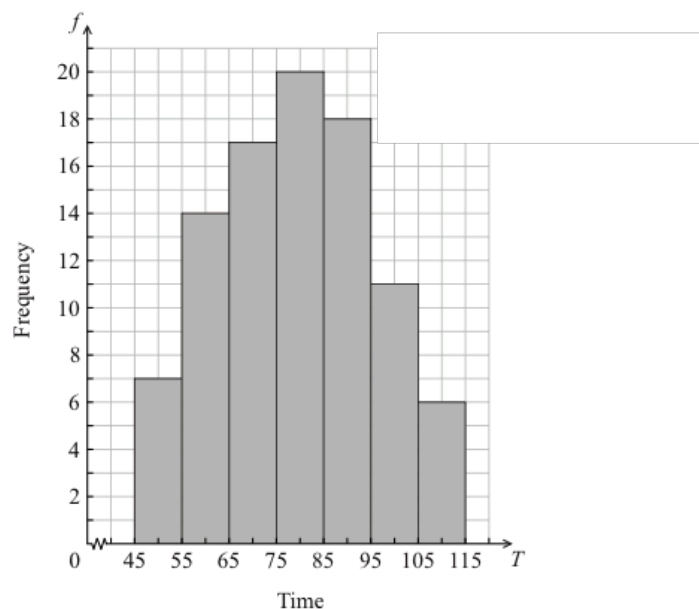
[4 marks]

(b) the fourth tail on the tenth toss.

[3 marks]

8. [Maximum mark: 13]

The histogram below shows the time T seconds taken by 93 children to solve a puzzle.



The following is the frequency distribution for T .

Time	$45 \leq T < 55$	$55 \leq T < 65$	$65 \leq T < 75$	$75 \leq T < 85$	$85 \leq T < 95$	$95 \leq T < 105$	$105 \leq T < 115$
Frequency	7	14	p	20	18	q	6

(a) (i) Write down the value of p and of q .

(ii) Write down the median class.

[3 marks]

(b) A child is selected at random. Find the probability that the child takes less than 95 seconds to solve the puzzle.

[2 marks]

Consider the class interval $45 \leq T < 55$.

(c) (i) Write down the interval width.

(ii) Write down the mid-interval value.

[2 marks]

(d) Hence find an estimate for the

(i) mean;

(ii) standard deviation.

[4 marks]

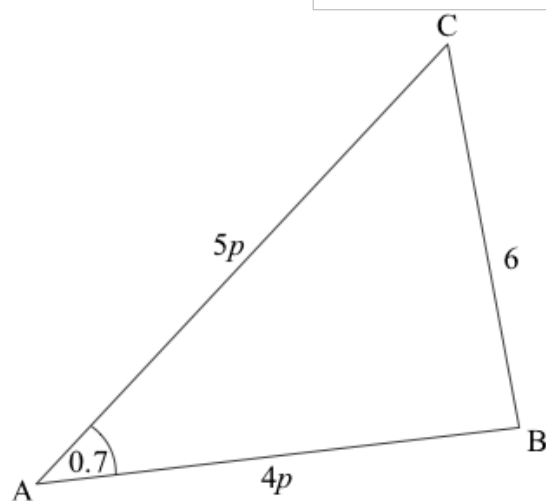
John assumes that T is normally distributed and uses this to estimate the probability that a child takes less than 95 seconds to solve the puzzle.

(e) Find John's estimate.

[2 marks]

9. [Maximum mark: 15]

The following diagram shows a triangle ABC



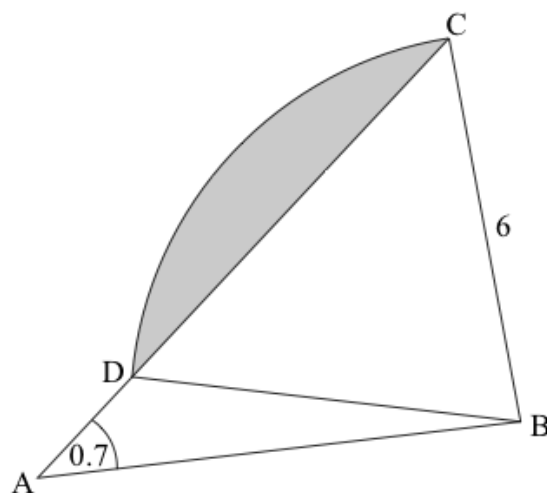
$BC = 6$, $\hat{CAB} = 0.7$ radians, $AB = 4p$, $AC = 5p$, where $p > 0$.

(a) (i) Show that $p^2(41 - 40 \cos 0.7) = 36$.

(ii) Find p .

[4 marks]

Consider the circle with centre B that passes through the point C. The circle cuts the line CA at D, and $\hat{A}DB$ is obtuse. Part of the circle is shown in the following diagram.



(b) Write down the length of BD.

[1 mark]

(c) Find $\hat{A}DB$.

[4 marks]

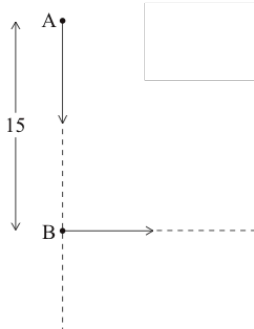
(d) (i) Show that $\hat{C}BD = 1.29$ radians, correct to 2 decimal places.

(ii) Hence, find the area of the shaded region.

[6 marks]

10. [Maximum mark: 17]

The following diagram shows two ships A and B. At noon, ship A was 15 km due north of ship B. Ship A was moving south at 15 km h^{-1} and ship B was moving east at 11 km h^{-1} .



(a) Find the distance between the ships

(i) at 13:00;

(ii) at 14:00.

[5 marks]

Let $s(t)$ be the distance between the ships t hours after noon, for $0 \leq t \leq 4$.

(b) Show that $s(t) = \sqrt{346t^2 - 450t + 225}$.

[6 marks]

(c) Sketch the graph of $s(t)$.

[3 marks]

(d) Due to poor weather, the captain of ship A can only see another ship if they are less than 8 km apart. Explain why the captain cannot see ship B between noon and 16:00.

[3 marks]